

# Game Theory and Applications

## Volume 6 (2001)

### Preface

#### Time-Consistent Solution for the Game of Information Trading ..... 1–8

*E. Eguiazarova*

#### Abstract

In this paper the questions are considered and examined for a special class of games which are called "games of information trading". It was in Driessen et al. (1992) that this class of games was introduced and a cooperative model of the game of information trading was proposed.

The present paper is devoted to the problem of time consistency in the game of information trading. As a solution concept the Shapley Value is considered. We show that the Shapley Value is time inconsistent in most cases. A regularization is proposed and the Shapley Value corresponding to the "regularized" c. f. is proved to be time consistent.

#### Game Problems for Systems with Volterra Evolution. Fractal Games ..... 9–44

*S.D. Eidelman and A.A. Chikrii*

#### Abstract

In this work we use the basic ideas of the method of resolving function to obtain sufficient conditions for solvability of the game problems. Minor assumptions are made about the dynamical process in order to encompass as wide range of conflict-controlled processes as possible. A number of fundamental methods exist in the theory of differential games that allow to formulate conditions for solvability of the problems of approach and avoidance in one or another class of strategies. Various mathematical techniques are used depending on the kind of exchange of information between the players about the process state and also on how the player, standing on which side the game is analyzed, chooses his control. In this paper as a tool for investigation was chosen the method of resolving functions based on using the inverse functionals of Minkowski and substantiating the classic rule of parallel pursuit. Under different forms of Pontryagin's condition this method was successfully applied in study of the game problems with groups of participants, games with the terminal functional, with state constraints and/or with imperfect information as well as in study of the processes with more complicated than ordinary differential equations dynamics. This paper is apparently one of the first attempts to study the game problems for system with fractional derivatives. Authors are also unaware of any existing examples of using the generalized matrix functions of Mittag-Leffler, defined in the paper and playing here an important role.

**Parlour Games with a Threshold ..... 45–52**

*A.Y. Garnaev*

**Abstract**

A new class of parlour games where the players are to maximize the payoffs depending on sum of hands so that this sum of hands not to exceed a threshold value is considered. One and two person games are investigated. The optimal strategies are found in closed form.

**Intergenerational Cooperative Solution of a Renewable Resource Extraction Game ..... 53–72**

*S. Jørgensen and D.W. Yeung*

**Abstract**

In this paper, we present an institutional structure for a cooperative intergeneration game of resource extraction. The government plays the role of an arbitrator and acts as a proxy for unborn individuals. Then we formulate an intergenerational equity rule for renewable resource extraction which maximizes recurrent resource rent and satisfies Solow's (1986, p.143) welfare criterion that each generation is as well off as any other generations. We designate the rule as the Maximum Recurrent Rent (MRR) Rule.

One of the most thorny issues in the debate on intergenerational equity is the absence of an implementation scheme that would be incentive compatible for rational agents. Here we show that the MRR rule can be implemented with an incentive scheme which includes a state-dependent reward, an extraction levy and an entry fee. Given this incentive scheme the maximum amount of recurrent resource rent can be achieved by rational behaviour of the extractors.

**On a Model of Two-card Poker ..... 73–81**

*V.V. Mazalov and I.S. Makhankov*

**Abstract**

The game has two players, each is dealt two cards:  $x_1, x_2$  to the first and  $y_1, y_2$  to the second player. After seeing his cards each player chooses one of the possible actions: announcing for "Lo" or for "Hi" depending on the value of the hands and independently of each other. Thus, when making the decision none of the players knows the choice of the opponent. The player who has announced for "Lo" in the case when his opponent announced for "Hi" loses one unit. If both players announce for "Hi" the hands are compared. If the lowest hand of one player is higher than the highest hand of the other player the former wins "A", if otherwise his highest hand is lower than the lowest hand of the opponent the former loses "A". In all other cases the game is a draw. The present paper dwells on two poker models: one with simultaneous moves (symmetric) and the other with the players making moves in turn (asymmetric).

**About one Pursuit Problem with Many Evaders ..... 82–88**

*N.N. Petrov*

**Abstract**

Sufficient condition and in some cases necessary conditions of the capture of even though one evader by the condition of the use by evaders the same control are derived. This paper is related to the investigations described by Satimov and Mavatov (1983), Petrov (1997).

**On Vector-Valued Markov Games ..... 89–100**

*M. Piškurič*

**Abstract**

The paper discusses discounted two-person zero-sum vector-valued Markov games. The model and the definition of equilibria are presented, and solution procedures, based on dynamic programming methods are explored.

**On a Solution of Cooperative Games under Uncertainty ..... 101–115**

*A.O. Remizov*

**Abstract**

In this paper a concept of guaranteed root-mean-square division in the cooperative games of two persons without side payment under uncertainty is considered. On the basis of the collective rationality principle a new concept of guaranteed root-mean-square division is stated, its properties and problems of existence and construction in some particular cases are investigated.

**Optimal Stopping Games where Players Have  
Weighted Privilege ..... 116–131**

*M. Sakaguchi*

**Abstract**

A non-zero-sum  $n$ -stage game version of a full-information best-choice problem under ENV maximization is analysed and solutions are obtained in some special cases of 2-person and 3-person games. The essential feature contained in this multistage game is the fact that the players have their own weights by which at each stage one player's desired decision is preferred to the opponent's one by drawing a lottery.

**Pure-Strategy Equilibrium in a Location Game  
with Discriminatory Pricing ..... 132–140**

*M. Sakaguchi*

**Abstract**

Consider a market of a homogeneous product by the two firms I and II. Customers in the market are distributed over the given segment  $[0, 1]$  with the given  $pdf f(\theta)$ ,  $0 \leq \theta \leq 1$ , and the firms I and II are located at the points  $x$  and  $y$ ,  $0 \leq x < y \leq 1$ , respectively. A competitive location game of  $H$ . Hotelling with simple discriminatory pricing is considered and pure-strategy equilibrium is derived. A special three-firm market is also discussed.

<b>Solution of the Repeated Prisoner's Dilemma with Finite Memory in Hierarchical Statement .....</b>	<b>141–163</b>
<i>A.A. Semenishchev</i>	

#### **Abstract**

A hierarchical statement of the infinitely repeated Prisoner's Dilemma with finite memory of the players is considered. An algorithm of constructing the optimal strategies is developed. It is shown that the profits of both the leader and the follower are greater than ones, obtained in the case of mutual defection of the players. For memory length one and two steps the optimal strategy of the leader, depending on the game parameters, has been constructed.

<b>Convergence of Learning Algorithms for Games on Networks</b>	<b>164–177</b>
<i>M.A. Sukhotina and L. A. Petrosyan</i>	

#### **Abstract**

The present research considers three different learning schemes for games on a given network. Each scheme is a dynamic procedure. At any stage of the procedure, a player comparing his payoff with payoffs of his neighbors can adopt behavior of the neighbor with the highest stage payoff. The learning schemes differ from each other only in definition of players stage payoffs (the maximal, the maxmin, or the average payoff determined by a bimatrix game). Given a 2x2 bimatrix game of a special type, it is shown that three learning procedures converge after a finite number of stages in a class of stationary strategies. Some results about the convergence are obtained for 3x3 bimatrix games.

<b>Subtraction Games .....</b>	<b>178–184</b>
<i>S.V. Vinnichenko</i>	

#### **Abstract**

The following game is considered. Let positive real numbers  $m, M, m \leq M$  and a nondecreasing function  $g(x)$  defined on  $]m, \infty[$  such that:  $g(m) \geq m, g(x) \leq g(m) + m$  for  $m \leq x < m + g(m)$  are given. Two players subtract in turn numbers from a positive real number  $X$ . In the beginning, the first player can subtract any real number  $x$  which satisfies restrictions  $m \leq x \leq M$ . On the next move, the second player can subtract any real number  $x'$  which satisfies restrictions  $m \leq x' \leq g(x)$ , etc. The game ends when the difference becomes non-positive and the player which doing this becomes a winner.

In the present paper we consider similar problem where the game ends at a negative difference and the second restriction on the function  $g$  is not required.

<b>Optimality Conditions for One-Shot Matrix Games .....</b>	<b>185–191</b>
<i>Tae-Hwan Yoon and O-Hun Kwon</i>	

#### **Abstract**

We find two optimality conditions for a one-shot undominated matrix game. The second optimality condition is formulated by Farkas' theorem. Using our condition, it is formally proved that there is no optimal solution in a one-shot  $(2 \times 2)$  undominated matrix game.

## **Auction Games and Integrative Imputations ..... 192–203**

*N.A. Zenkevich*

### **Abstract**

The basic auction game with divisible good and its applications are presented. The Nash equilibrium in mixed strategies for basic two-person auction model is constructed. The main application has been shown for cooperative game. New optimality principle so-called integrative imputation and optimal integrative imputation (OI-imputation), is constructed. These imputations exist for all considered cooperative games, but the OI-imputation is unique. The important property of the imputation is that it is supported by Nash equilibrium in the corresponding auction game.